Wind power – Problem

The offshore wind farm at ”Lillgrund”, south off the Öresund-bridge, consists of 48 HAWT wind turbines. It is estimated that a total of 330 GWh_{el} per year will be generated by the wind farm. This is enough for 60,000 households.

The turbines, which are of model MKII from Siemens Wind Power AB, are 115 m high at the highest point of the rotating tips. The Rotor diameter is 93m and the rotational speed varies from 6 -16 rpm. The mean wind speed at “Lillgrund” is 8-10 m/s.

Determine the total axial force, the torque, the power and the power coefficient off a MKII wind turbine. (Hint: see Example 10.7 in Dixon). Use the following parameters when calculating with the BEM method:

Air density	ρ = 1.2 Kg/m^3
Wind speed	$c_{x1} = 9$ m/s
Constant lift coefficient [a]	C_L = 0.7

[a] This is a design strategy that leads to constant incidence angles, α, if we neglect the Reynolds number dependence (since then $C_L = f(\alpha)$).

Chord length [b] ℓ = 2.5 m (or c = 2.5)
Rotational speed N = 12 rpm

Divide the blade into 9 sections (see Example 10.7 in "Dixon"). The "active" part of the radius can be assumed to range from r/R=0.1 -1.

a) Compute the power output and power coefficient, C_P. Plot the pitch angle, β, as a function of r (or r/R). You should obtain β = .122 radians (approx.) for r/R = 0.55. Plot the velocity tringles at root (small r/R), mid-span and near tip.
b) With the above computed pitch angles, compute the incidence angles if the turbine is operated the same way at c_{x1} = 13 m/s. Compute the power coefficient. How can it be increased?

Below matlab loop solves the case with predefined incidence (α) given the variables

- Omega (scalar speed of rotation in rad/s)
- rho (density kg/m3)
- cx1 (axial cvelocity upstream of turbine, m/s)
- R; (rotor radius in meters)
- J (tip speed ratio)
- Z (number of blades)
- c (chord length, m)
- num_el (number of elements)
- rR (r/R, dimensionless vector length num_el)
- alfa (incidence, vector length num_el)

and using the functions CL_fit and eps_fit

Optional for interested students:

c) Correct the solution from a) for finite number of blades using Prandtl's correction (eq 10.44b, 10.43 and 10.46). Again, compute the power output and power coefficient.

[b] Constant chord length is clearly not the way modern wind turbines are designed, see note on solid mechanics.

```
for i=1:num_el

    % initiate
    a(1)=0;
    ap(1)=0;

    phi(1)=atan(1/rR(1)/J*(1-a(1))/(1+ap(1)));  % eq 10.39

    %     alfa(1)=phi(1)-beta;
    beta(1)=phi(1)-alfa(1);
    CL(1)=CL_fit(alfa(1));
    CD(1)=CD_fit(CL(1));


    % start iteration
    jmax=2000;
    j=1; res=1e4;
    while (j<jmax) && (abs(res)>1e-8)
        j=j+1;
        % RHS of eq. 10.31
        var=Z*c*(CL(j-1)*cos(phi(j-1))+CD(j-1)*sin(phi(j-1)))/(8*pi*R*rR(i)*sin(phi(j-1))^2);   %RHS of eq. 10.31
        a(j)=var/(var+1);  % solve for axial induction factor
        var=Z*c*(CL(j-1)*sin(phi(j-1))-CD(j-1)*cos(phi(j-1)))/(8*pi*R*rR(i)*sin(phi(j-1))*cos(phi(j-1))); %RHS of eq 10:33
        ap(j)=var/(1-var);  % solve for tangential induction factor
        phi(j)=atan(1/rR(i)/J*(1-a(j))/(1+ap(j)));  % eq 10.39
        beta(j)=phi(j)-alfa(i);
        CL(j)=CL_fit(alfa(i));
        CD(j)=CD_fit(CL(j));
        res=(a(j)-a(j-1))/a(j);  % rel residual
    end
    if ([j/jmax]==1), warning('convergence criteria not reached'), end

    dtau(i)=4*pi*rho*Omega*cx1*(1-a(j))*ap(j)*(rR(i)*R).^3; %eq 10.16b
    Delta_tau(i)=0.5*rho*Z*c*Omega^2*R^4*((1+ap(j))/cos(phi(j)))^2*rR(i)^3*CL(j)*sin(phi(j))*drR; % eq 10.42
    dP(i)=dtau(i)*Omega;

    % somehow save last iteration a(j),ap(j) and angles
    out(i,1:6)=[rR(i),a(j),ap(j),alfa(i),beta(j),phi(j)];

end % for i=1:num_el
```

Lift and drag

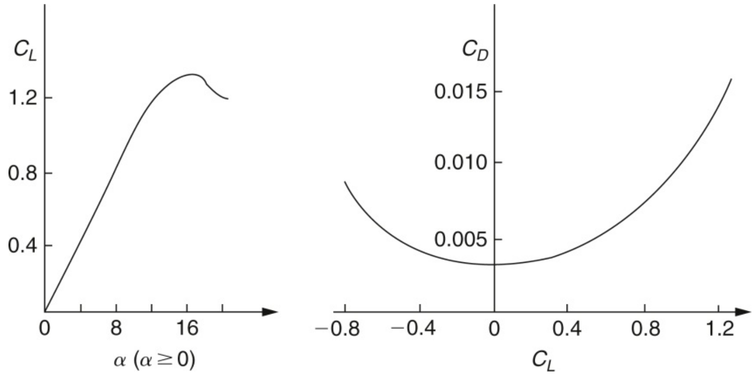

Figure 10.13

```
function CL=CL_fit(alfa) % in radians
if (alfa > 0.352) ||  (alfa < -.136)
    error('alfa out of range')
else
%     CL=2*pi*sin(alfa);
%     CL=0.7;

    CL = 2062.74*alfa^6 - 1160.26*alfa^5 +33.403*alfa^4 + 24.904*alfa^3 - 1.2013*alfa^2 + 5.9293*alfa;

end
end

function CD=CD_fit(CL) % note NOT alfa
CD = 2.66e-3*CL^4-2.32E-03*CL^3 + 6.00E-03* CL^2 + 7.12E-04*CL + 0.00315;
end
```

Solid mechanics

- The thrust on the entire rotor results in a torque that the mast has to withstand.
- Also the blades are bent by the distributed axial force upon them. Sometimes the rotating axis is not really horizontal, but slightly inclined so that the risk for the blades to collide with the tower is reduced.
- On top of this we have centrifugal forces

This leads to

- Tapering; the blade chord is reduced at the ends (amongst other) to reduce centrifugal forces and axial bending of the blade
- Very thick blades (high t/l) at smaller radii